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# Simplified Transformer Models with Their Loads and Distributed Energy Resources for Three-Phase Power Flow Calculation in Unbalanced Distribution Systems

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**Abstract**

This paper proposes the simplified models of distribution transformer with their loads and distributed energy resources(DERs) for solving three-phase power flow in unbalanced distribution systems. Instead of the rigorous transformer models for power flow analysis, the simplified models can dramatically improve the convergence problem; especially in modern power distribution systems that interconnected with DERs. The simplified models, which considering the transformer connections and their loads and DERs are derived in this paper. The common transformer connections, such as single-phase, open wye, open delta and three-phase are included. After rigorously proofed the accuracy of these models, the proposed models are applied to calculate the individual phase complex power at each high voltage bus. Finally, the proposed models are applied to a sample distribution system for solving three-phase power flow. The outcomes demonstrate that the proposed models can reduce the nodes of system and furthermore simplified the system structure; therefore, the convergence problem of power flow can be improved.

Keyword: Power Distribution System, Three-Phase Power Flow, Simplified Model, Distributed Energy Resources, Load Flow.

## 1. Introduction

Recently, the major function of modern power distribution system is distinct from the traditional power system, due to more and more DERs have been interconnected into distribution networks. Therefore, the distribution systems are not only to deliver electric power to customers, but also have DERs to generate electric power to customers and even more reversed power to transmission systems. Generally, the DERs are composed of generation systems and energy storage devices[1]. Where the generation systems are included renewable and non-renewable units, and the energy storage devices are consist of super inductor, super capacitor, and fly wheeling. The power distribution systems that includes DERs and controllable/uncontrollable loads can be operated in grid-tied and islanding modes with a single point of common coupling are called microgrids[2-3]. Additionally, the microgrids that use the information and communication technology to gather and act on information to improve the efficiency, reliability, economics, and sustainability of the systems are called smart grid[4]. Consequently, the use of microgrids is the new development trend in power systems; therefore, investigating the structure of the system, related operating control technology, and scheduling management is necessary. Especially, the unbalanced power flow analysis is essential to the planning, design, and operation stages in power distribution systems. For rigorous analysis of power distribution systems, detailed solution techniques have been developed[5-7]; however, the convergence problem still exist. In which the asymmetrical winding connections of distribution transformers are the major factor that results in convergence problem. In this paper, in order to improve the efficiency and convergence problems, the new simplified models of distribution transformers with their loads and DERs are derived base on the developed models[8]. Instead of rigorous transformer models, the derived models are used in power flow analysis by the OpenDSS software[9]. Numerical results demonstrate that this simplified approach can improve the efficiency and convergence problems with high solution accuracy.

## 2. Derivation of Simplified Models and Their Applications

### 2.1 Concept of Simplified Models

Power transformer is one of the important components in power systems. This device can change AC electric power at one voltage level to another voltage level. In general, various distribution transformer connections, such as single-phase, open wye-open delta, open delta-open delta, and three-phase connections are used in a modern distribution system to meet the requirements of different customers.

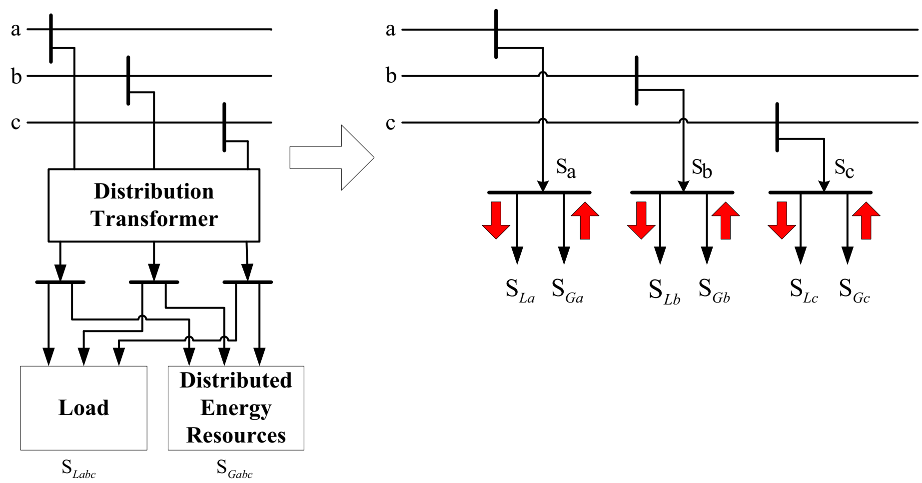


Figure 1 Schematic diagram of the proposed integrated models

In this section, a distribution transformer with its loads and DERs can be integrated and simply represented by its equivalent loads and generations as shown in Figure 1. The equivalent individual complex power are functions of the connection and loads and generations of the distribution transformer. The transformers are assumed to be ideal for simplifying the derivation procedure. The equivalent individual phase complex power of three common transformer connections with their loads and generations are derived as follows.

### 2.2 Line-to-ground transformer supplying a load and a DER

Figure 2 shows the circuit diagram of a line-to-ground transformer supplying a load and a DER, where a, b, and c represented phase a, b, and c, respectively. It is obviously that the single-phase load and generation of the DER are supplied only by the phase a of the source; therefore, the individual phase complex power at the primary side of the transformer can be represented as (1).

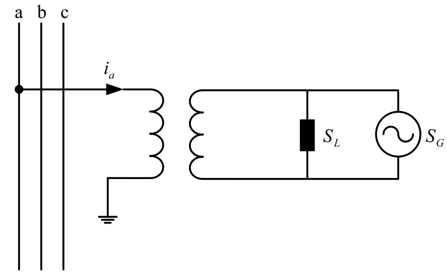


Figure 2 Line-to-ground transformer supplying a load and a DER

$$\begin{cases} S_a = S_L - S_G \\ S_b = 0 \\ S_c = 0 \end{cases} \tag{1}$$

### 2.3 Grounded wye-delta transformer supplying various loads and DERs

The circuit diagram of a line-to-ground transformer supplying various loads and DERs is shown in Figure 2. The individual phase complex power at the primary side of the transformer can be derived as shown in (2).

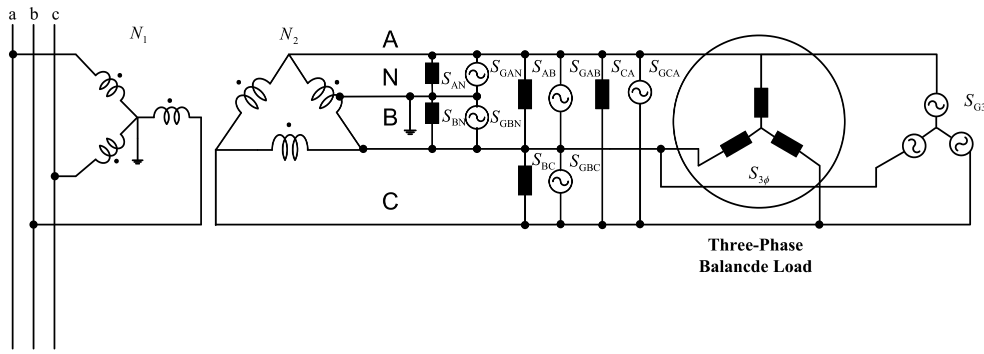


Figure 3 Grounded wye-delta transformer supplying various loads and DERs

$$
\begin{cases}
S_a = \left[ \dfrac{2(S_{AN}+S_{BN})}{3} + \dfrac{2S_{AB}}{3} + \dfrac{S_{BC}}{3}\angle -60^\circ + \dfrac{S_{CA}}{3}\angle 60^\circ + \dfrac{S_{3\phi}}{3} \right] \\
\quad - \left[ \dfrac{2(S_{GAN}+S_{GBN})}{3} + \dfrac{2S_{GAB}}{3} + \dfrac{S_{GBC}}{3}\angle -60^\circ + \dfrac{S_{GCA}}{3}\angle 60^\circ + \dfrac{S_{G3\phi}}{3} \right] \\
S_b = \left[ \dfrac{S_{AN}+S_{BN}}{3}\angle 60^\circ + \dfrac{S_{AB}}{3}\angle 60^\circ + \dfrac{2S_{BC}}{3} + \dfrac{S_{CA}}{3}\angle -60^\circ + \dfrac{S_{3\phi}}{3} \right] \\
\quad - \left[ \dfrac{S_{GAN}+S_{GBN}}{3}\angle 60^\circ + \dfrac{S_{GAB}}{3}\angle 60^\circ + \dfrac{2S_{GBC}}{3} + \dfrac{S_{GCA}}{3}\angle -60^\circ + \dfrac{S_{G3\phi}}{3} \right] \\
S_c = \left[ \dfrac{S_{AN}+S_{BN}}{3}\angle -60^\circ + \dfrac{S_{AB}}{3}\angle -60^\circ + \dfrac{S_{BC}}{3}\angle 60^\circ + \dfrac{2S_{CA}}{3} + \dfrac{S_{3\phi}}{3} \right] \\
\quad - \left[ \dfrac{S_{GAN}+S_{GBN}}{3}\angle -60^\circ + \dfrac{S_{GAB}}{3}\angle -60^\circ + \dfrac{S_{GBC}}{3}\angle 60^\circ + \dfrac{2S_{GCA}}{3} + \dfrac{S_{G3\phi}}{3} \right]
\end{cases}
\tag{2}
$$

## 2.4 Open wye-open delta transformer supplying various loads and DERs

Figure 3 shows the circuit diagram of an open wye-open delta transformer with mid-tap grounded at one leg on the secondary side supplying various loads and DERs. The individual phase complex power at the primary side of the transformer can be easily obtained as shown in (3).

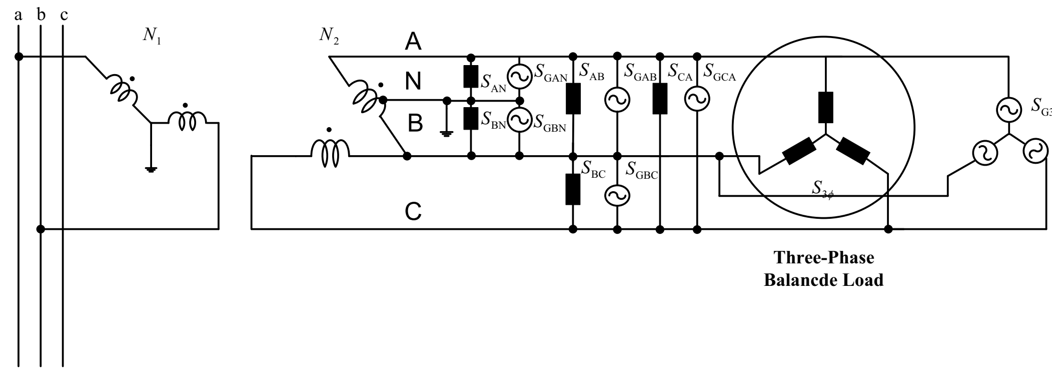


Figure 4 Open wye-open delta transformer with mid-tap grounded at one leg on the secondary side supplying various loads and DERs

$$
\begin{cases}
S_a = \left[ S_{AN} + S_{BN} + S_{AB} + S_{CA}\angle 60^\circ + \dfrac{S_{3\phi}}{\sqrt{3}}\angle 30^\circ \right] \\
\quad - \left[ S_{GAN} + S_{GBN} + S_{GAB} + S_{GCA}\angle 60^\circ + \dfrac{S_{G3\phi}}{\sqrt{3}}\angle 30^\circ \right] \\
S_b = \left[ S_{BC} + S_{CA}\angle -60^\circ + \dfrac{S_{3\phi}}{\sqrt{3}}\angle -30^\circ \right] \\
\quad - \left[ S_{GBC} + S_{GCA}\angle -60^\circ + \dfrac{S_{G3\phi}}{\sqrt{3}}\angle -30^\circ \right] \\
S_c = 0
\end{cases}
\tag{3}
$$

## 3. Three-Phase Power Flow Analytic Techniques with DERs

The Distribution System Simulator (DSS) development began in 1997 at Electrotek Concepts, Inc. This program has advanced over the years and obtained many new capabilities. These capabilities are quite useful for analyzing DERs issues. In 2008, Electric Power Research Institute (EPRI) made the program open source to support grid modernization efforts by providing researchers and consultants with a tool to evaluate progressive ideas; therefore, the DSS was renamed OpenDSS. In this paper, the OpenDSS is used to analyze three-phase power flow of unbalanced distribution systems with the proposed integrated models. Without rigorous mathematical models of distribution transformers, these integrated models can improve the convergence problem and enhance the efficiency.

## 4. Numerical Results and Discussions

### 4.1 Description of the Sample System

Figure 5 shows the single line diagram of the sample system. There are 15 distribution transformers with different rating and connection types. According to various capacities of each DER, they are interconnected into the secondary side of distribution transformers, respectively. The power generations of the DERs are assumed as constant power model in this paper. Consequently, the proposed simplified models are used to calculated the individual phase complex power at the primary side of each transformer, and the computing results are shown in Table 1. The calculated complex power at each bus of the primary side of each transformer is adopted to simulate the three-phase power flow by OpenDSS.

Table 1 Individual phase complex power calculated by integrated models

| Tr No. | Phase A | | Phase B | | Phase C | |
|---|---|---|---|---|---|---|
| | kW | kvar | kW | kvar | kW | kvar |
| **1** | 56.15 | 82.74 | 99.74 | -7.25 | 0 | 0 |
| **2** | 0 | 0 | 45.83 | 107.3 | 115.84 | 13.96 |
| **3** | 96 | 72 | -7.18 | 59.57 | 55.18 | -23.57 |
| **4** | 85 | 52.68 | -1.56 | 49.98 | 44.06 | -23.64 |
| **5** | 59.09 | 110.15 | 124.94 | 3.9 | 0 | 0 |
| **6** | 155.83 | 96.58 | -2.86 | 91.63 | 80.78 | -43.33 |
| **7** | 160 | 120 | -11.96 | 99.28 | 91.96 | 39.28 |
| **8** | 64.83 | -4.72 | 0 | 0 | 36.5 | 53.79 |
| **9** | 65.57 | 106.42 | 124.95 | -3.58 | 0 | 0 |
| **10** | 0 | 0 | 39.38 | 73.41 | 83.27 | 2.6 |
| **11** | 64.83 | -4.72 | 0 | 0 | 36.5 | 53.79 |
| **12** | 0 | 0 | 13.61 | 31.86 | 0 | 0 |
| **13** | 0 | 0 | 39.29 | 57.9 | 69.79 | -5.08 |
| **14** | 91.64 | 2.86 | 0 | 0 | 43.34 | 80.79 |
| **15** | 99.28 | 11.96 | 0 | 0 | 39.28 | 91.96 |
| **Total** | **998.22** | **645.95** | **464.18** | **564** | **696.5** | **240.55** |

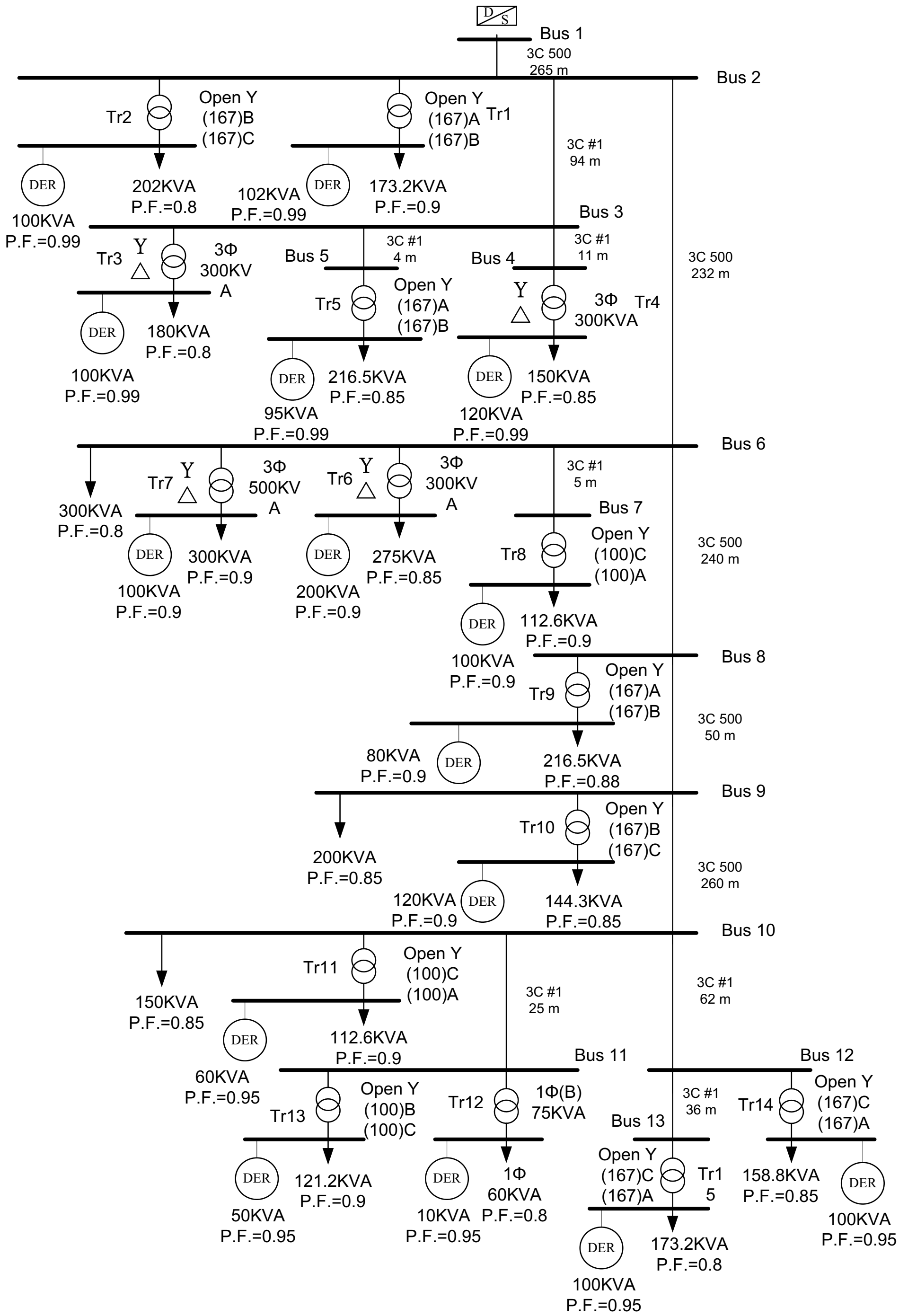


Figure 5 Single line diagram of the sample system

### 4.2 Simulation Results and Discussions

In this paper, the detailed simulation results of the rigorous transformer models are not presented due to the length limit. It is sure that the mismatches of voltage profiles between the rigorous transformer models and the proposed simplified models are less than $10^{-3}$. The simulation results of bus voltage profiles are shown in Table 2. The numerical results reflect the physical phonemona of this system that includes the asymmetrical feeder structure, transformer connections, loads, and power generations of DERs. Moreover, the simulation results of the complex power and current flows in each line section are shown in Table 3. From the numerical results, it is definite that the DERs can supply the electrical power to the loads neraby; therefore, the loading of each transformer is reduced and then the line flow in each line section is lessened. Besides, the computing time required for the proposed models in power flow solution is less than that of rigurous transformer models. To sum up, the proposed models adopted in three-phase power program can really improve the solution efficiency and convergence problems.

Table 2 Simulation results of bus voltage profiles

| Bus No. | Phase A (pu) | Phase B (pu) | Phase C (pu) |
|---|---|---|---|
| **Bus1** | 1.00000 | 1.00000 | 1.00000 |
| **Bus2** | 0.99866 | 0.99898 | 0.99890 |
| **Bus3** | 0.99818 | 0.99879 | 0.99870 |
| **Bus4** | 0.99809 | 0.99889 | 0.99863 |
| **Bus5** | 0.99800 | 0.99848 | 0.99870 |
| **Bus6** | 0.99762 | 0.99812 | 0.99792 |
| **Bus7** | 0.99760 | 0.99812 | 0.99790 |
| **Bus8** | 0.99725 | 0.998778 | 0.99753 |
| **Bus9** | 0.99722 | 0.99775 | 0.99749 |
| **Bus10** | 0.99696 | 0.99749 | 0.99710 |
| **Bus11** | 0.99696 | 0.99735 | 0.99701 |
| **Bus12** | 0.99660 | 0.99749 | 0.99696 |
| **Bus13** | 0.99638 | 0.99749 | 0.99690 |

Table 3 Simulation results of complex power and current flows profiles

| Line section \ Phase | Phase A | | Phase B | | Phase C | |
|---|---|---|---|---|---|---|
| | $S_a$ (kW+jkvar) | $I_a$ (A) | $S_b$ (kW+jkvar) | $I_b$ (A) | $S_c$ (kW+jkvar) | $I_c$ (A) |
| **Bus 1-Bus 2** | 741+j657 | 50 | 581+j489 | 38 | 593+j554 | 41 |
| **Bus 2-Bus 3** | 52+j181 | 10 | 42+j8 | 2 | 31+j47 | 3 |
| **Bus 2-Bus 6** | 678+j428 | 41 | 493+j410 | 32 | 498+j506 | 36 |
| **Bus 3-Bus 4** | 6+j42 | 2 | 21+j6 | 1 | 7+j24 | 1 |
| **Bus 3-Bus 5** | 16+j76 | 4 | 74+j0 | 4 | 0 | 0 |
| **Bus 6-Bus 8** | 295+j118 | 16 | 222+j142 | 13 | 163+j237 | 15 |
| **Bus 6-Bus 7** | 7+j0 | 0 | 0 | 0 | 4+j6 | 0 |
| **Bus 8-Bus 9** | 255+j50 | 13 | 143+j144 | 10 | 163+j237 | 15 |
| **Bus 9-Bus10** | 255+j50 | 13 | 143+j128 | 10 | 149+j237 | 14 |
| **Bus 10-Bus 11** | 0 | 0 | 16+j49 | 3 | 21+j0 | 1 |
| **Bus 10-Bus 12** | 96-j29 | 5 | 0 | 0 | 13-j130 | 7 |
| **Bus 12-Bus 13** | 61-j29 | 3 | 0 | 0 | 18-j92 | 5 |

## 5. Conclusions

In this paper, the new simplified models of distribution transformers with their loads and DERs are proposed and are adopted in three-phase power flow program. The OpenDSS is used to execute the unbalanced power flow of the sample system that is composed of asymmetrical structures of feeder and transformer, and the loads and DERs. The outcomes demonstrate that the proposed models can reduce the nodes of system and furthermore simplified the system structure. Consequently, the convergence problem and the efficiency of power flow can be improved. Furthermore, the simplified power flow calculation approach is helpful for the planning, operation, and control in modern distribution systems with different types of loads and DERs.

## 6. Acknowledgment

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